Exam 2 2049H Spring 2009

Solns

Name:

Scores: **1**

2

3

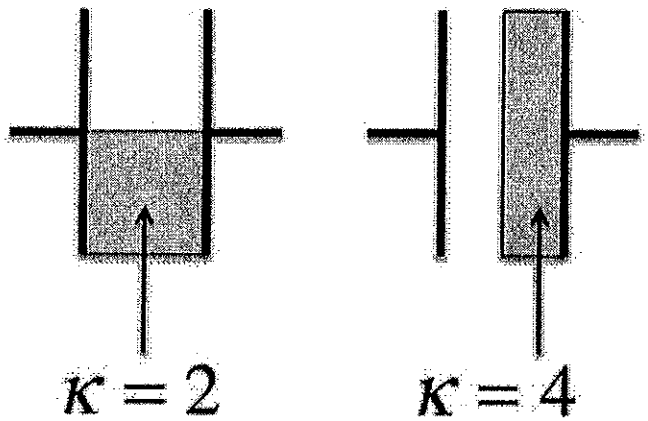
4

5

Total:

Problem 1 (15 points)

Parallel capacitors are half filled by dielectric materials as shown below. (a) calculate the capacitances in terms of ε_0, A, d. (b) Which capacitor has higher capacitance?



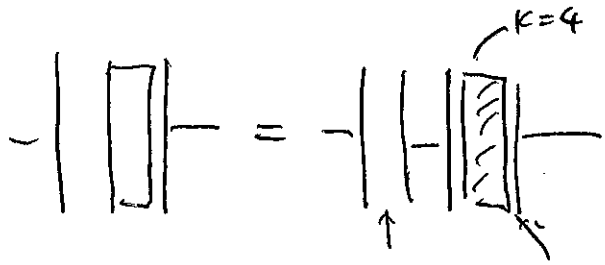
b) RIGHT CAPACITOR HAS HIGHER CAPACITANCE

a) $C = \frac{\varepsilon_0 A}{2d}$ PARALLEL

$C_T = \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_0 A}{d}$

$C = \frac{\varepsilon_0 A/2}{2d}$ ($\kappa = 2$)

$C_T = \frac{3}{2}\frac{\varepsilon_0 A}{d} = 1.5\frac{\varepsilon A}{d}$



$C = \frac{2\varepsilon_0 A}{d}$ $\quad C = \frac{8\varepsilon_0 A}{d}$

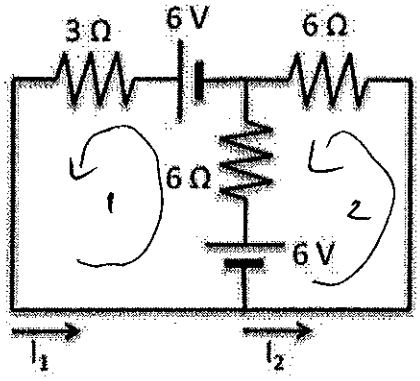
$\frac{1}{C_T} = \frac{1}{\frac{2\varepsilon_0 A}{d}} + \frac{1}{\frac{8\varepsilon_0 A}{d}}$

$= \frac{d}{\varepsilon_0 A}\left[\frac{1}{2} + \frac{1}{8}\right] = \frac{d}{\varepsilon_0 A}\left[\frac{4+1}{8}\right]$

$C = \frac{8}{5}\frac{\varepsilon_0 A}{d} = 1.6\frac{\varepsilon_0 A}{d}$

Problem 2 (20 points)

Calculate I_1 and I_2 for the circuit shown left.



$12 - \frac{27}{2} + \frac{63}{42}$

Loop 1 $\quad \Delta V = 6V - 6(I_1 - I_2) + 6 - 3I_1$

$= 12 - 6I_1 + 6I_2 - 3I_1$

$= 12 - 9I_1 + 6I_2 = 0 \quad \ldots (1)$

Loop 2 $\quad \Delta V = -6I_2 - 6(I_2 - I_1) - 6 = 0$

$-12I_2 + 6I_1 = 6 \quad \ldots (2)$

(1) : $12 = 9I_1 - 6I_2$

(2) : $6 = 6I_1 - 12I_2$

(2) $\times \frac{3}{2}$: $9 = 9I_1 - 18I_2 \quad ; (3)$

(1) − (3) $\Rightarrow \quad 3 = 0 + 12I_2$

$\boxed{I_2 = 1/4\,A}$

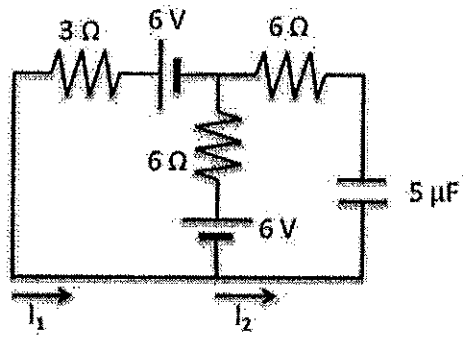
PLUG BACK INTO (2)

$-3 + 6I_1 = 6$

$6I_1 = 9$

$\boxed{I_1 = 3/2\,A}$

Problem 3 (30 points)

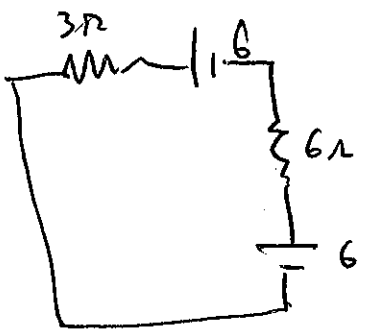


(a) Calculate I_1 in the steady state (t=∞). Note that at this point I2 should be equal to zero.

(b) Calculate the charge stored in the capacitor in the following circuit.

(c) What is the characteristic charging time (the RC time constant) for the capacitor?

a) IF $I_2 = 0$ CIRCUIT LOOK LIKE



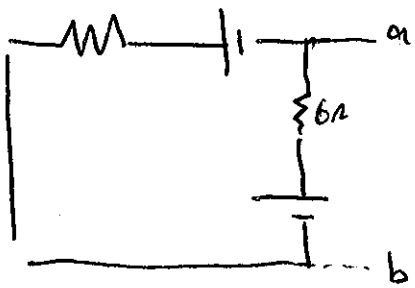
$6 - 6I_1 + 6 - 3I_1 = 0$

$12 = 9I_1$

$\frac{4}{3} = I_1$

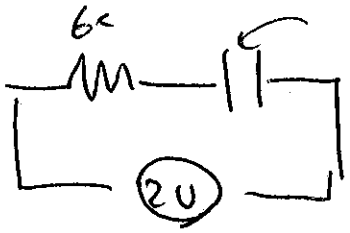
$I_1 = \frac{4}{3}$ A

b) IF $I_1 = 4/3$



$V_{ab} = -\frac{4}{3} \cdot 6 + 6$

$= -2$ VOLTS



$V_{CAPACITOR} = 2V$

$CV = Q$

$5 \times 10^{-6} \cdot 2V = Q = 1 \times 10^{-5}$ C

C) RC TIME CONSTANT IS

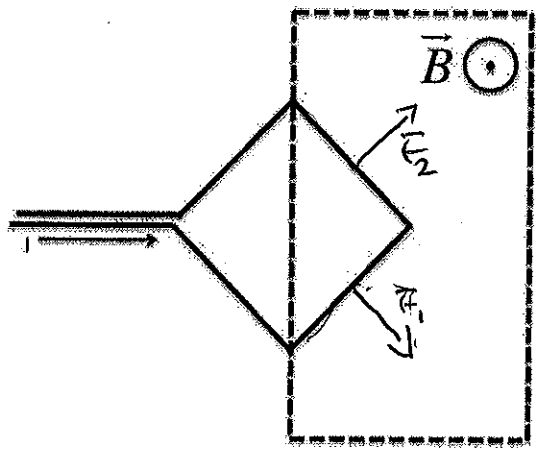
$R = 6\Omega$

$C = 5\mu F$

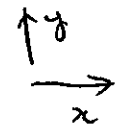
$RC = (6\Omega)(5\mu F) = 30 \times 10^{-6}$ sec

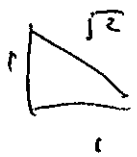
$= \boxed{30\mu s}$

Problem 4 (20 points)



Calculate the force due to magnetic field on a wire which has been shaped into a square shape as shown left. The square has sides of 1 m, the magnetic field is 0.1 T, and the current is 1 A. Magnetic field is only present in the dotted square as indicated in the figure.





$$|\vec{F}_1| = |\vec{F}_2| = |I\vec{L} \times \vec{B}| = 1A \cdot 1m \cdot 0.1T$$

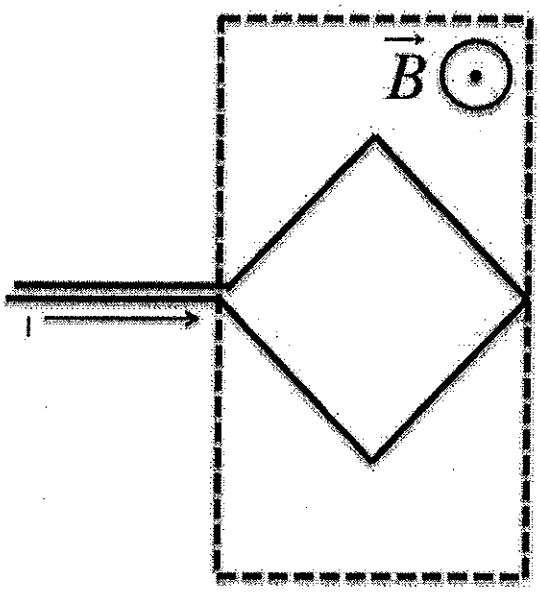
$$= \boxed{0.1N}$$

BUT FORCES IN y DIRECTION CANCEL AND ONLY FORCES IN x DIRECTION ADD

$$F_T = 0.1N \frac{2}{\sqrt{2}} = 0.1 \frac{2\sqrt{2}}{2}$$

$$= \boxed{0.141 \cdot N} \text{ IN x DIRECTION}$$

Problem 5 (15 points)



Calculate the force if the metal square has been inserted further into the magnetic field. Everything else remains the same. The square has sides of 1 m, the magnetic field is 0.1 T, and the current is 1 A. Magnetic field is only present in the dotted square as indicated in the figure.

ALL FORCES CANCEL

F = 0